Sub-Planck Structures in Phase Space and Heisenberg-Limited Measurements

Diego A. R. Dalvit and Wojciech H. Zurek, T-QC; and Luiz Davidovich and Fabricio Toscano, Rio de Janeiro

Quantum measurement was, for a long time, synonymous with the foundations of quantum theory. Yet, this subject has recently acquired practical, engineering relevance, as it is now known that using probes prepared in judiciously chosen quantum states can increase their sensitivity. As a consequence, quantum metrology became a subject of great practical interest. Typical problems in quantum metrology are high-precision phase measurements (for example, in quantum interferometry), and the detection of weak forces (such as signal in gravitational wave detectors). The precision of the measurement of these small perturbations depends on the resources involved in the measurement process, such as the total number of photons employed, the type of probes used for the detection, the efficiency of the detectors, etc. Using probes prepared in quasiclassical states, such as coherent states of light, the precision is at the standard quantum limit (SQL): the minimal detectable phase shift is inversely proportional to the square root of the number of photons used. By employing highly nonclassical quantum states (such as Schrödinger cat states) it is possible to surpass the SQL limit and reach the ultimate precision allowed by quantum theory: the Heisenberg limit. Here the minimal detectable phase shift scales as the inverse of the number of photons. Achieving such a limit would open exciting possibilities for ultrasensitive detection of weak forces and other quantum technology applications [1].

Every quantum state can be represented in phase space by means of the Wigner distribution, which is the closest analog to the classical phase space representation of a system. The Wigner function of a nonclassical quantum state, such as superpositions of coherent states (Fig. 1), possesses small-scale structures that are surprisingly small [2]: they have phase space areas much smaller than the Planck size one would naively guess for the size of phase space features from the Heisenberg uncertainty principle. These sub-Planck structures represent interference effects. Typical area of such structures can be as small as (but no smaller than) Planck constant *divided* by the classical action associated with the state (which is proportional to the number of photons contained in that state). When a perturbation acts on such a nonlocal state, it will typically rotate or displace it in phase space. The perturbed and unperturbed states can be distinguished from one another (they become quasiorthogonal) when the peaks (valleys) of the sub-Planck structures of one come on top of the valleys (peaks) of the sub-Planck structures of the other (Fig.1). Thus, the size of the sub-Planck structures sets a sensitivity limit on a probe prepared initially in such a state [2]. The typical linear size of the sub-Planck structures is inversely proportional to the square root of the number of photons contained in the nonclassical state. Therefore, states that saturate the limit on the smallest phase space structures can allow one to attain Heisenberg-limited sensitivity.

We have developed [3] two experimental proposals to implement these ideas with cold atomic systems, both in the context of cavity quantum electrodynamics (QED) and trapped ions. In the cavity QED experiment, one would entangle the state of the electromagnetic field inside the cavity with the internal electronic states of an atom that flies through the cavity. By appropriately choosing the interaction time between the two systems it is possible to create a "Schrödinger cat" state of the electromagnetic field (i.e., a superposition of two coherent states of light) that, when subjected to a perturbation, has Heisenberg-limited sensitivity. The information of the perturbation is encoded in the atomic state, so that by measuring the populations of the atom as it exits the cavity, one can retrieve information of the perturbation. The ion trap experiment has more flexibility, and it is possible to create more complicated correlated states, such as a superposition of four coherent states (a "compass state") of the vibrational motion of the ion in the trap. Such states have sub-Planck structures, and are therefore useful to Heisenberg-limited precision measurements. Compass states can be generated using quantum state engineering by tailoring the interaction between external lasers and the internal and external dynamics of the trapped ion. This provides us with different operations for doing controlled coherent quantum dynamics, such as conditional rotations and translations. These and other operations form a toolbox that is required, not only for doing ultrasensitive detection, but are also employed for quantum information processing and quantum simulation with cold atomic systems.

For more information contact Diego A. R. Dalvit at dalvit@lanl.gov.

[1] V. Giovannetti, et al., *Science* **306**, 1330 (2004).
[2] W.H. Zurek, *Nature* (London) **412**, 712 (2001).
[3] D.A.R. Dalvit and W. Zurek, "Heisenberg-limited Measurement of Displacements and Rotations with Cavity QED and Ion Traps," Los Alamos National Laboratory report LA-UR-05-4229 (June 2005). F. Toscano, et al., quant-ph/0508093.

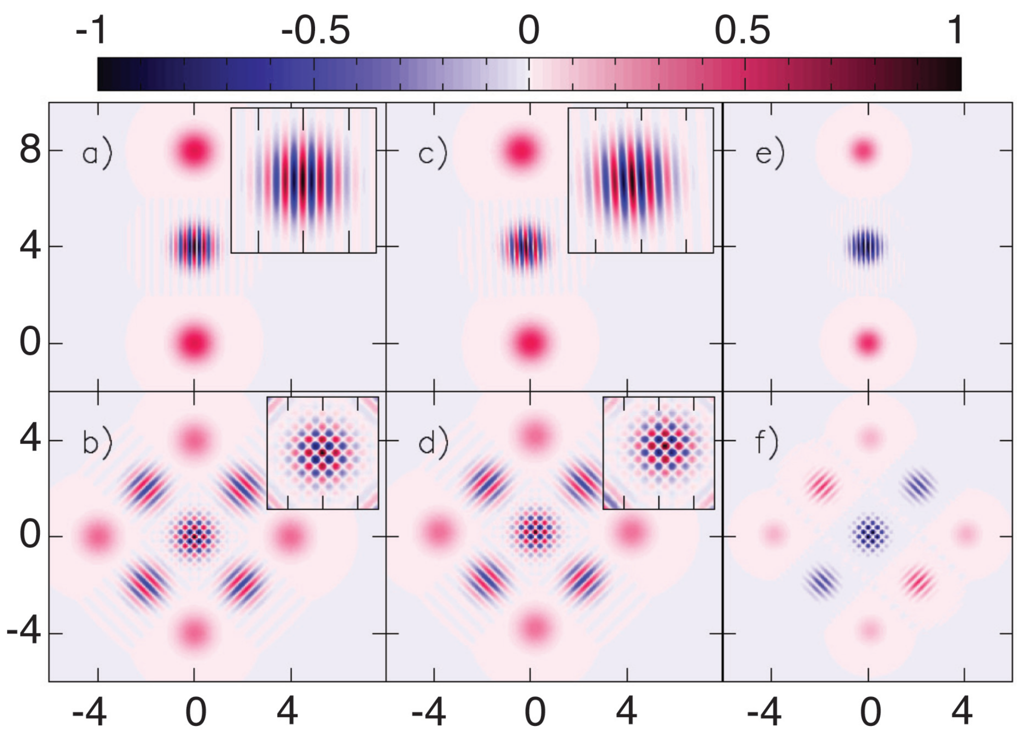

Fig. 1. ***Correlated quantum states have sub-Planck structures in their Wigner function that are closely related to Heisenberg-limited sensitivity. Panels a) and c) show a cat state (i.e., the superposition of two coherent states) before and after a rotation is applied to the system. Panels b) and d) show a compass state (i.e., the superposition of four coherent states) before and after a displacement is applied to the system. In e) and f) we display the respective products of the unperturbed and perturbed Wigner functions. When performing the integration over phase space, the negative contributions (in blue) cancel the positive ones (in red), leading to quasiorthogonality. The minimal detectable rotations and displacements are at the Heisenberg limit.***